Optimization and games in energy markets

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Outline

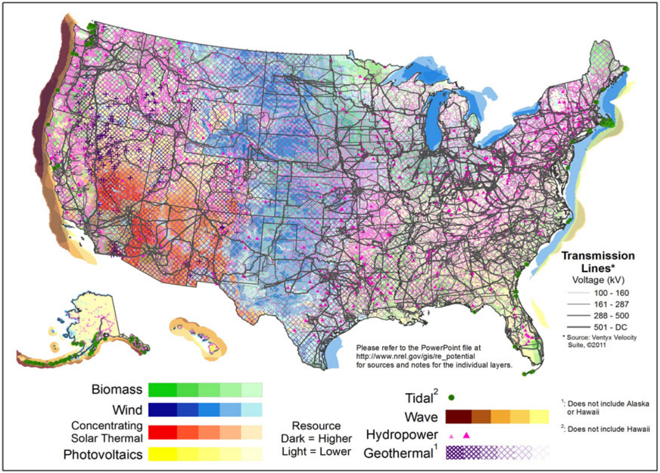
- Introduction
- Whole sale markets and Equilibrium. Discontinuous Games
- Nash and beyond
- Mechanism design and market Power
- Efficient regulations and Extended Mechanism Design
- Conclusions

1. Introduction and motivation
2. Modeling Market
   - Equilibrium: Nash
3. Intrinsic Market Power
4. Efficient regulations and mechanism design
   - The benchmark game
   - Comparing Benchmark with Optimal Mechanism

Motivations

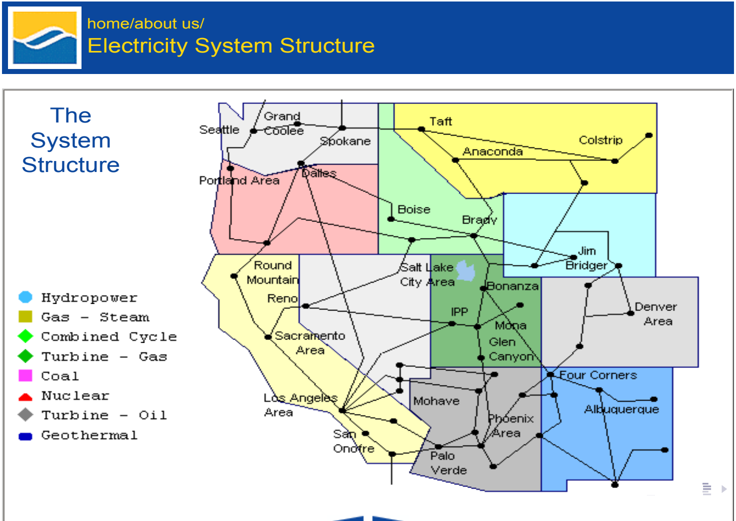
- Most of ISOs have few generation companies: oligopoly
- Transmission networks highly congested in some areas
- Intrinsic market power produced by externalities and information asymmetries

Transmission system USA

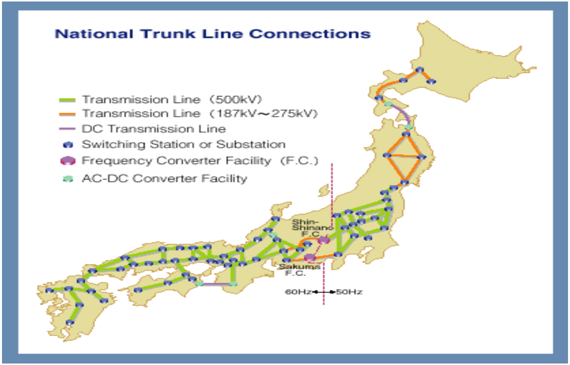


Transmission California

Electricity System Structure

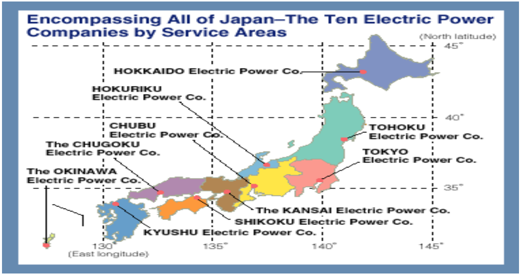


Transmission Japan



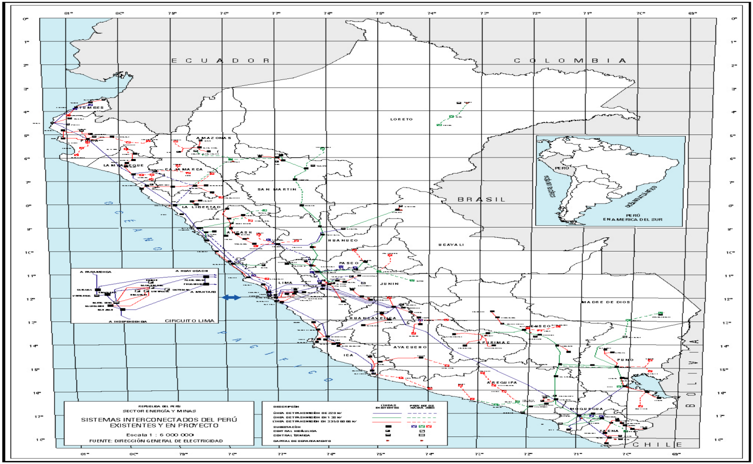
Generation system Japan

companies.pdf



Transmission Chile

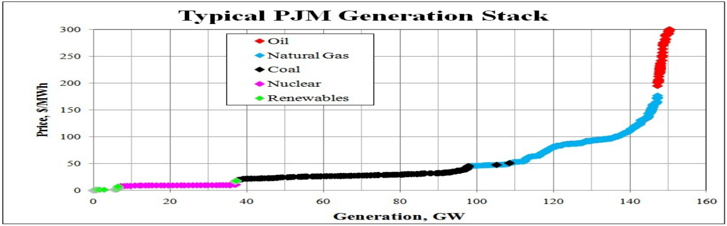
Transmission Peru



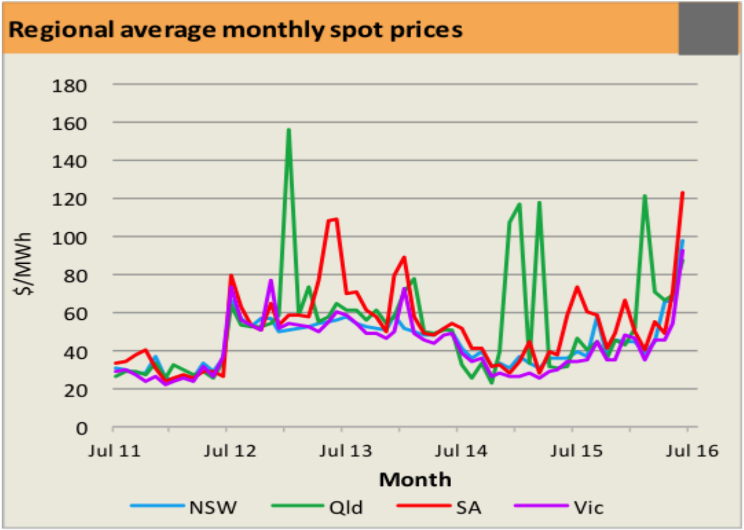
What determines the price of electricity? example

- Between 50 % and 60 % of the price of electricity is the price at which generators sell power to the wholesale electricity market; this element of the price is determined by competition. In a new framework of massive entry of renewal energy that is changing.
- The cost of transmitting and distributing electricity accounts for another 30 % and includes system charges to cover network investment and operation as determined by the Commission for Energy Regulation.
- The remaining 10 % to 15 % is supplier charges and government taxes and levies, such as the Public Service Obligation (PSO), which cover services relating to creating a secure and sustainable electricity system.

Generation costs



Prices Australia Regional



A generation short term market: day-ahead mandatory pool

- Today: generators taking into account an estimation of the demand simultaneously bid *convex real-valued nondecreasing*. In applications often nondecreasing piece-wise linear cost functions or equivalently piece-wise constant ”price”.
- Tomorrow: the (ISO) using this information and knowing a realization of the demand, minimizes the sum of the costs to satisfy demands at each node considering all the transmission constraints: ”dispatch problem”.
- Tomorrow: the (ISO) sends back to generators the optimal quantities and ”prices” (multipliers associated to supply = demand balance equation at each node)

ISO problem or dispatch $DP(c, d)$

The (ISO) knows a realization of the demand $d \in \mathbb{R}^V$, receives the costs functions bid $(c_i)_{i \in G}$ and compute: $(q_i)_{i \in G}$, $(\lambda_i)_{i \in G}$

$$\min_{(h,q)} \quad \sum_{i \in G} c_i(q_i). \tag{1}$$

$$\sum_{e \in K_i} \frac{r_e}{2} h_e^2 + d_i \leq q_i + \sum_{e \in K_i} h_e sgn(e, i), \quad i \in G \tag{2}$$

$$q_i \in [0, \bar{q}_i], \quad i \in G, \tag{3}$$

$$0 \leq h_e \leq \overline{h}_e \tag{4}$$

We denote $Q(c,d) \subset \mathbb{R}^G$ the generation component of the optimal solution set associated to each cost vector submitted $c = (c_i)$ and demand d.
We denote $\Lambda(c,d) \subset \mathbb{R}^G$ the set of multipliers associated to the supply=demand in the ISO problem.

Modeling Generators

1. At each node $i \in G$ we have a generator with payoff

$$u_i(\lambda, q) = \lambda q - \bar{c}_i(q)$$

   $\bar{c}_i$ is the real cost.

2. The strategic set for each player i denoted X_i:

   $\{c_i : \mathbb{R} \to \mathbb{R}_+ |$ convex, nondecreasing, bounded subgradients $\}$

   $\partial c_i \subset [0, p^*]$, p^* is a *price cap*.

Applications: nondecreasing piece-wise linear

Equilibrium

An equilibrium is (q, λ, m) such that q is a selection of $Q(\cdot, \cdot)$ and λ is a selection of $\Lambda(\cdot, \cdot)$ and $m = (m_i)_{i \in G}$ is a mixed-strategy equilibrium of the generator game in which each generator submits costs $c_i \in S_i$ with a payoff

$$\mathbb{E}u_i(\lambda_i(c, \cdot), q_i(c, \cdot)) = \int_D [\lambda_i(c, d) q_i(c, d) - \bar{c}_i(q_i(c, d))] d\mathbb{P}(d),$$

Neutral or risk averse

Remark: this game is played everyday !

Full problem

$$\min_{(h,q)} \quad R[\sum_{i\in G} c_i(q_i) + \gamma r(\bar{q})] \tag{5}$$

$$\sum_{e\in K_i} \frac{r_e}{2} h_e^2 + d_i \leq q_i + \sum_{e\in K_i} h_e sgn(e,i), \quad i \in G \quad <-- \quad \lambda \tag{6}$$

$$\lambda \geq 0 \text{ (or any sign}^2\text{)} \tag{7}$$

$$q_i \in [0, \bar{q}_i], \quad i \in G, \quad 0 \leq h_e \leq \overline{h}_e \tag{8}$$

$$\forall \omega \quad or \quad chance \quad or \quad cvar \quad constraints \tag{9}$$

$$\max_{(c_i)} \quad \mathbb{E} u_i(\lambda_i(c_i, c_{-i}), q_i(c_i, c_{-i})) = \int_D [\lambda_i(c,d) q_i(c,d) - \bar{c}_i(q_i(c,d))] d\mathbb{P}(d$$

[2] Germany, Belgium, ..

Nash equilibrium

Consider a game $G = (X_i, u_i)^N$ that consists of N players where each player $i = 1, \ldots, N$ has a strategy set X_i and a payoff function $u_i : X \to \mathbb{R}$, where $X = \Pi_{i \in N} X_i$.

Nash equilibrium (x_i^*)

$$x_i^* \in \text{ argmax } \{u_i(x_i, x_{-i}^*) | \, x_i \in X_i\}$$

Nash equilibrium

For the sake of simplicity, we assume that each X_i is contained in a metric vectorial space:

- If for all i the strategy set X_i is a compact set, and u_i is a bounded function, we say that G is a *compact game*.
- If for all i the set X_i is convex and for each $x_{-i} \in X_{-i}$, $u_i(\cdot, x_i)$ is a (concave) quasiconcave function, then we say that G is a *convex game* (*quasiconvex game*)

Nash equilibrium existence

A convex compact game $G = (X_i, u_i)^N$ satisfying:

- $u_i(\cdot, \cdot)$ is upper semicontinuous
- $u_i(x_i, \cdot)$ is lower semicontinuous for all x_i

has a Nash equilibrium point.

Extensions: generalized games, convergence-stability lopsided convergence

Discontinuos games: tie-breaking rules

Consider the following two-player game: Let the payoff for the i player be given by

$$u_i(x_i, x_{-i}) = \begin{cases} l_i(x_i) & \text{if} \quad x_i < x_{-i}, \\ \varphi(x_i) & \text{if} \quad x_i = x_{-i}, \\ m_i(x_{-i}) & \text{if} \quad x_i > x_{-i}, \end{cases} \tag{10}$$

where $x_i \in [0,1]$. Assume that for all i and $x \in [0,1]$ (a) l_i and m_i are continuous functions, l_i is nondecreasing $\varphi(x)$ is a convex combination of $l_i(x)$ and $m_i(x)$;
sign $[l_i(x) - \varphi(x)] =$ sign $[\varphi_{-i}(x) - m_{-i}(x)]$.

Existence discontinuos games

Reny (1999) Econometrica

Theorem

A compact quasiconcave game possesses a Nash equilibrium if it is also a better reply secure game.

Bagh and J. (2006) Econometrica

Theorem

If $(X_i, u_i)^N$ is weakly reciprocally upper semicontinuous and payoff secure, then it is better reply secure.

Full problem

$$\min_{(h,q)} \quad R[\sum_{i \in G} c_i(q_i) + \gamma r(\bar{q})] \tag{11}$$

$$\sum_{e \in K_i} \frac{r_e}{2} h_e^2 + d_i \leq q_i + \sum_{e \in K_i} h_e sgn(e,i), \quad i \in G \quad < - - \quad \lambda \tag{12}$$

$$\lambda \geq 0 \text{ (or any sign}^3\text{)} \tag{13}$$

$$q_i \in [0, \bar{q}_i], \quad i \in G, \quad 0 \leq h_e \leq \overline{h}_e \tag{14}$$

$$\forall \omega \quad or \quad chance \quad or \quad cvar \quad constraints \tag{15}$$

$$\max_{(c_i)} \quad \mathbb{E}u_i(\lambda_i(c_i, c_{-i}), q_i(c_i, c_{-i})) = \int_D [\lambda_i(c,d) q_i(c,d) - \bar{c}_i(q_i(c,d))] d\mathbb{P}(d$$

[3] Germany, Belgium, ..

Assumptions

S1. For all $d \in D$, there exists $\delta_d > 0$ such that

$$\Omega(d) \neq \emptyset, \quad \|\hat{d} - d\| \leq \delta_d.$$

S2. D is compact
S3. (1) Either $\mathbb{P}$ is non atomic; or (2) given two convex sets $M, N \subset \mathbb{R}^G$, $u(M \times N)$ is convex.
S4 $u_i : \mathbb{R}^2 \to \mathbb{R}$ is continuous.

Equilibrium existence

Proposition: $\Lambda(c,d)$ is nonempty and the image is bounded by

$$\lambda^* = 2p^* \frac{\sum \bar{q}_v}{\delta} \in \mathbb{R}_+.$$

Theorem

If each S_v is a nonempty closed set for the point-wise convergence, then there exists an equilibrium (q, λ, m) selection-mixed strategy for the bid-based generator pool game.

Example: In real system... increasing piece-wise constant cost functions

Idea of proof

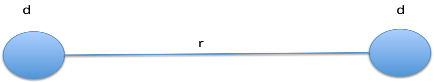
- We can endow each set of strategies with the epi-metric, then X_i become metric and compact.
- Prove that $EU : X \to R^G$ has closed graph, bounded range and is convex-valued
- Lemma (Simon and Zame [1990]) Under these conditions there exists a (measurable) selection $V \in EU$ such that the normal form game $(X_i, V_i)_{i \in G}$ possesses a mixed strategy Nash equilibrium $\bar{m}$, which is a measure over the Borel epi-convergence $\sigma - field$.

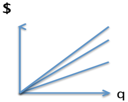
Two-node case

Two nodes case

Symmetric Nash equilibrium

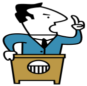
Profit = multiplier × quantity − cost × quantity





Nash = c/(1-2rd)

d < 1/2r



the ISO Problem: two-node case

Given that each generator reveals a cost c_i, the (ISO) solves:

$$\begin{array}{ll} \min\limits_{q,h} & \sum\limits_{i=1}^{2} c_i q_i \\ s.t. & q_i - h_i + h_{-i} \geq \frac{r}{2}[h_1^2 + h_2^2] + d \ \ \text{for} \ \ i = 1, 2 \\ & q_i, h_i \geq 0 \ \ \text{for} \ \ i = 1, 2 \end{array}$$

Result

- Escobar and J. (ET (2010)) equilibrium exists but producers charge a price above marginal cost:
- $$Nash = \bar{c}/(1 - 2rd)$$

Market Power formula

Proposition

The equilibrium prices p_i satisfy

$$\mathbb{E}|p_i - \gamma| \geq \frac{\mathbb{E}[Q_i(p_i, p_{-i}, d)]}{\bar{\eta}_i}$$

where $\bar{\eta}_i = 2\frac{|K_i|^2\left(1+\max\{r_e\overline{h}_e\ :\ e\in K_i\}\right)^2}{p_* \min_{e\in K} r_e}$

$\gamma(p_{-i}, d)$ *is a measurable selection of* $\partial\bar{c}_i(Q_i(p_i, p_{-i}, d))$.

Market Power formula

Proposition

Linear case: $\bar{c}_i(q) = \bar{c}_i q$, *then*

$$p_i - \bar{c}_i \geq \frac{\mathbb{E}[Q_i(p_i, p_{-i}, d)]}{\bar{\eta}}.$$

The Questions

In an electric network with **transmission costs** and **private information**:

- Does the usual (price equal Lagrange multiplier) regulation mechanism minimize costs for the society?
- If not, what is the mechanism that achieves this objective?
- How does the performance of both systems compare?

Methodology:

- Bayesian Game Theory
- Mechanism Design

Framework

- A network with demand d at each node.
- One producer at each node, with piece-wise linear cost of production $c_i \sim F_i[\underline{c}_i, \overline{c}_i]$. Common knowledge ! This game is played everyday !
- Transmission costs rh^2, with h the amount sent from one node to another.

ISO for piece-wise linear cost functions

Problem

$$\underset{(q,h)}{\textit{minimize}} \quad \sum_{i=1}^{n}\sum_{j=1}^{N} q_{i,j}c_{i,j}$$

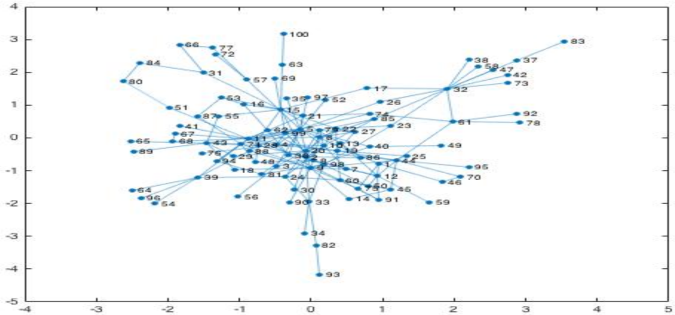
$$\sum_{j=1}^{N} q_{i,j} + \sum_{i' \in V(i)} h_{i',i} - h_{i,i'} - \frac{h_{i,i'}^2 + h_{i',i}^2}{2} r_{i,i'} \geq d_i \quad (\lambda_i)$$

$$\forall (i,i') \in E : h_{i,i'} \geq 0 \quad (\gamma_{i,i'})$$

$$\forall i \in I, j \in J : q_{i,j} \geq 0 \quad (\mu_{i,j})$$

$$\forall i \in I, j \in J : q_{i,j} \leq \bar{q} \quad (\nu_{i,j}). \tag{16}$$

100 nodes network



The ISO Problem: two-node case

Given that each generator reveals a cost c_i, the ISO solves:

$$
\begin{aligned}
\min_{q,h} \quad & \sum_{i=1}^{2} c_i q_i \\
s.t. \quad & q_i - h_i + h_{-i} \geq \tfrac{r}{2}[h_1^2 + h_2^2] + d \quad \text{for} \quad i = 1, 2 \\
& q_i, h_i \geq 0 \quad \text{for} \quad i = 1, 2
\end{aligned}
$$

The Solution for ISO problem

If we define

$$H(x,y) = d + \frac{1}{2r}\left(\frac{x-y}{x+y}\right)^2 - \frac{1}{r}\left(\frac{x-y}{x+y}\right)$$

and

$$\overline{q} = 2\left[\frac{1-\sqrt{1-2dr}}{r}\right]$$

then the solution to this problem can be written as

$$q_i(c_i, c_{-i}) = \begin{cases} H(c_i, c_{-i}) & \text{if } H(c_i, c_{-i}) \geq 0 \text{ and } H(c_{-i}, c_i) \geq 0 \\ \overline{q} & \text{if } H(c_{-i}, c_i) < 0 \\ 0 & \text{if } H(c_i, c_{-i}) < 0 \end{cases}$$

$$\lambda_i(c_i, c_{-i}) \equiv p_i(c_i, c_{-i}) = c_i \quad \text{if } H(c_i, c_{-i}) \geq 0$$

The Bayesian Game: benchmark

The game:

- 2 players. Strategies $c_i \in C_i = [\underline{c}_i, \overline{c}_i]$, i=1,2.
- Payoff $u_i(c_i, c_{-i}) = (\lambda_i(c_i, c_{-i}) - \mathbf{c}_i) q_i(c_i, c_{-i})$,

where $\mathbf{c}_i$ is the real cost. The Equilibrium:

- A strategy $b_i : [\underline{c}_i, \overline{c}_i] \longrightarrow I\!R^+$ (convex at equilibrium!)
- In a Nash equilibrium

$$\overline{b}(c) \in arg\max_x \int_{C_{-i}} [\lambda_i(x, \overline{b}(c_{-i})) - c] q_i(x, \overline{b}(c_{-i})) f_{-i}(c_{-i}) dc_{-i} \tag{17}$$

Numerical Approximation

- For simplicity $C_i = [1, 2]$.
- Let $k \in \{0, ..., n-1\}$, and $b(c) = b_k$ for $c \in [\frac{k}{n}, \frac{k+1}{n}]$.
- The weight of each interval is given by $w_k = F(\frac{k+1}{n}) - F(\frac{k}{n})$.
- The approximate equilibrium is characterized by:

$$b_k \in arg\max_x \sum_{l=0}^{n-1} [\lambda_i(x, b_l) - r_k] q_i(x, b_l) w_l \quad \text{for all} \quad k \in \{0, ..., n-1\} \tag{18}$$

Optimal Mechanism. Principal Agent Model (Myerson)

- A *direct revelation mechanism* $M = (q, h, x)$ consists of an *assignment rule* $(q_1, q_2, h_1, h_2) : C \longrightarrow R^4$ and a *payment rule* $x : C \longrightarrow R^2$.
- The ex-ante expected profit of a generator of type c_i when participates and declares c_i' is

$$U_i(c_i, c_i'; (q, h, x)) = E_{c_{-i}}[x_i(c_i', c_{-i}) - c_i q_i(c_i', c_{-i})]$$

- A mechanism (q, h, x) is feasible iff:

$$\begin{aligned}
U_i(c_i, c_i; (q, h, x)) &\geq U_i(c_i, c_i'; (q, h, x)) \quad \text{for all} \quad c_i, c_i' \in C_i \\
U_i(c_i, c_i; (q, h, x)) &\geq 0 \quad \text{for all} \quad c_i \in C_i \\
q_i(c) - h_i(c) + h_{-i}(c) &\geq \frac{r}{2}[h_1^2(c) + h_2^2(c)] + d \quad \text{for all} \quad c \in C \\
q_i(c), h_i(c) &\geq 0 \quad \text{for all} \quad c \in C
\end{aligned}$$

The Regulator's Problem

Using the revelation principle, the regulator's problem can be written as:

$$\min \int_C \sum_{i=1}^{2} x_i(c) f(c) dc \tag{19}$$

subject to (q, h, x) being "feasible"

Existence: Knuster-Tarski fixed point theorem (monotone relations)

The Regulator's Problem (II)

It can be rewritten as

$$\begin{array}{ll} \min & \int_C \sum_{i=1}^{2} q_i(c)[c_i + \frac{F_i(c_i)}{f_i(c_i)}] f(c) dc \\ \text{s.t} & \int_{C_{-i}} q_i(c_i, c_{-i}) f_{-i}(c_{-i}) dc_{-i} \ \text{ is non-increasing in } \ c_i \\ & q_i(c) - h_i(c) + h_{-i}(c) \geq \frac{r}{2}[h_1^2(c) + h_2^2(c)] + d \ \text{ for all } \ c \in C \\ & q_i(c), h_i(c) \geq 0 \ \text{ for all } \ c \in C \end{array}$$

We denote by $J_i(c_i) = c_i + \frac{F_i(c_i)}{f_i(c_i)}$ the virtual cost of agent i. We assume it is increasing (Monotone likelihood ratio property: true for any log concave distribution)

Solution

An optimal mechanism is given by

$$\hat{q}_i(c_i, c_{-i}) = \begin{cases} H(J_i(c_i), J_{-i}(c_{-i})) & \text{if } H(J_i(c_i), J_{-i}(c_{-i})) \geq 0 \text{ ar} \\ \overline{q} & \text{if } H(J_{-i}(c_{-i}), J_i(c_i)) < 0 \\ 0 & \text{if } H(J_i(c_i), J_{-i}(c_{-i})) < 0 \end{cases}$$

$$\hat{x}_i(c_i, c_{-i}) = c_i \hat{q}_i(c_i, c_{-i}) + \int_{c_i}^{\overline{c}_i} \hat{q}_i(s, c_{-i}) ds$$

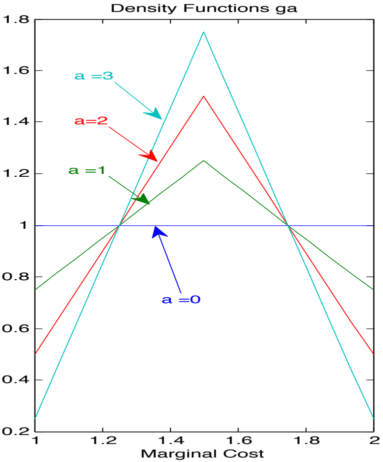
Such mechanism is dominant strategy incentive compatible.

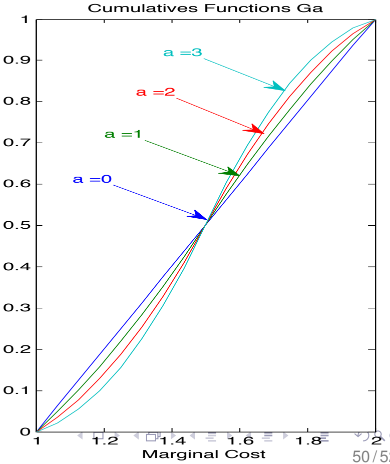
Comparing Benchmark with Optimal Mechanism

We consider the family of distributions with densities

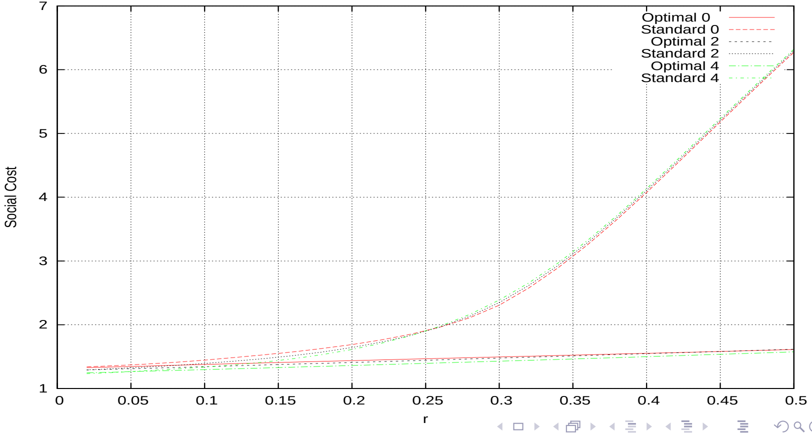
$$f_a(x) = \begin{cases} a(x-1) + (1 - \frac{a}{4}) & \text{if } \; x \leq 1.5 \\ -a(x-1) + (1 + \frac{3a}{4}) & \text{if } \; x \geq 1.5 \end{cases}$$

Asymmetric information





Social costs for different mechanisms



Robustness and Practical Implementation

- The optimal mechanism is detail free. If the designer is wrong about common beliefs, then the mechanism is still not bad:

$$||X_f - X_{\tilde{f}}|| \leq ||x||_1 ||f - \tilde{f}||_\infty \leq \bar{c}\bar{q}||f - \tilde{f}||_\infty$$

- The assignment rule is computationally simple to implement. It requires solving **once** the dispatcher problem, with modified costs.
- However, the payments are computationally difficult

$$c_i \hat{q}_i(c_i, c_{-i}) + \int_{c_i}^{\bar{c}_i} \hat{q}_i(s, c_{-i}) ds$$

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